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How can one estimate the *posteriori* distribution ?

$\Rightarrow$ sample according to this posterior distribution

- direct **sampling**
- **Markov chain Monte Carlo** (MCMC)

Monte Carlo method

Monte Carlo : von Neumann & Ulam
(*Los Alamos Scientific Laboratory* – 1955)

⇒ use random numbers to compute quantities whose analytical computation is hard (or impossible)

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- **Law of Large Numbers** (LLN)
- so-called "**Monte Carlo sample**"

$\Rightarrow$ compute various functions from that sample distribution

Example : One wants to compute $\mathbb{E}[f(X)] = \int f(x) p_X(x) dx$

If $x_i \overset{iid}{\sim} p_X$, $\mathbb{E}[f(X)] = \frac{1}{N}\sum_{i=1}^{N} f(x_i)$ (LLN)

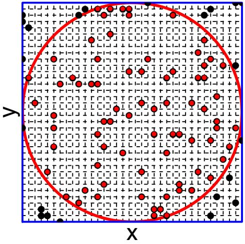
$\Rightarrow$ if one knows how to sample from p_X, one can then estimate $\mathbb{E}[f(X)]$

...

Monte Carlo method: illustration

π estimation:

A casino *roulette* (in Monte Carlo ?)



A 36×36 grid

1. The probability of being inside the disk while in the square: $p_C = \frac{\pi R^2}{(2R)^2} = \frac{\pi}{4}$
2. n points $\{(x_{11}, x_{21}), \dots, (x_{1n}, x_{2n})\} = \{P_1, \dots, P_n\}$ on the 36×36 grid (generated with the *roulette*)
3. Count the number of points inside the disk

⇒ Compute the ratio (estimated probability of being inside the disk while in the square): $\widehat{p}_C = \frac{\sum P_i \in circle}{n}$

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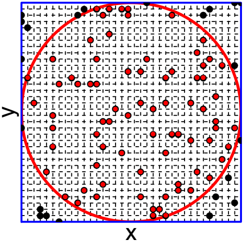

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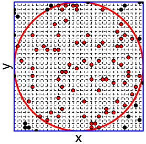

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If $n = 1000$ and 786 points are inside the disk : $\widehat{\pi} = 4 \times \frac{786}{1000} = 3.144$

One can improve the estimate by increasing:

- the **grid resolution**, and also
- the number of points sampled n: $\lim_{n \to +\infty} \widehat{p}_C = p_C = \pi/4$ (LLN)

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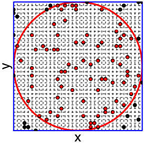

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Monte Carlo sample ⇒ compute various functions
e.g. $\pi = 4 \times$ the probability of being inside the disk

Your turn !

Practical: exercise 2